A REVIEW ON MATHEMATICAL ANALYSIS OF PAEDIATRIC DENTAL ARCH WITH NORMAL OCCLUSION

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ABSTRACT

Many mathematical models were proposed in the literature to define the form of the human dental arch. This paper review the literature to compare and analyse such dental studies with various mathematical models.

Key Words: Dental Arch; Mathematical Analysis; Normal Occlusion

In the dental literature a lot of effort has been made to define ideal form of the human dental arch. In the early literature the shape of the arch was considered to be elliptic, parabolic, and U-shaped and were inadequate to define the dental arch. The advancement in orthodontics in clinical and research process increase the need to discuss the case and develop treatment plan in multidisciplinary settings with an absolute demand to quantify arch forms. The conventional linear measurements such as arch width, depth, and circumference used to describe arch shapes are insufficient in characterization of arch form because they provide an incomplete description of all arch traits.

Various studies in the medical literature shows that physical appearances have a direct impact on the overall well being and inter-personal behaviour of the human individual. The deciduous dentition needs to be kept under observation for early detection and correction of any developing malocclusion in order to achieve a proper and healthy functioning permanent dentition. Deciduous canine, incisor and molar, plays a vital role in the development of final occlusion of permanent dentition.1 The arch dimensions has an important role in aligning teeth, stabilizing the form, alleviating arch crowding, and providing for a normal overbite and overjet, stable occlusion and a balanced facial profile.1

Arch size has been seen to be more important than teeth size in clinical diagnosis and a proper treatment planning required study of dental arch forms, shape, size and other parameters like overjet and overbite, along with spacing in deciduous dentition.2 In the past decades different mathematical models were formulated to describe of dental arch in humans with terms like elliptic, parabolic, etc. Many authors considered the arch circumference, width and depth as some of the relevant parameters for measuring the dental arch curve. They defined the dental arch curvature through use of biometry by measurement of ratios, angles and linear distances.3-10 These models have limitations in describing dental arch three-dimensionally (3D).11 Thus, there are numerous mathematical models and geometrical forms that have been put forth by various experts, but no two models appear to be clearly defined by means of a single parameter.10 Many mathematical models were proposed in the literature to define the form of the human dental arch. This paper review the literature to compare and analyse such dental studies with various mathematical models.

Dental Arch: In general the dental arch was described based on dental arch curvature include conic sections,12,13 parabolas,14 cubic spline curves,15 catenary curves16 and polynomials of second to eight degree17 mixed models and the beta function.8 Conic sections which are 2nd order curves, can only be applied to specific shapes like hyperbolas, ellipse, etc and their efficiency as ideal fit to any shape of the dental arch is thus limited.18 The beta function, although superior, considers only the parameters of molar width and arc depth and does not factor in other dental landmarks. Nor does it consider asymmetrical forms. In contrast, the 4th order polynomial functions are better effective in defining the dental arch than either cubic spline or the beta function.18 Some authors define human dental arch using mathematical modelling like symmetry or asymmetry.18

Occlusion: Occlusion is the manner in which the lower and upper teeth intercuspate between each other in all mandibular positions or movements. It is a result of neuromuscular control of the components of the mastication systems.19 The difference between physiological and pathological occlusion is that in physiological occlusion various components function smoothly and without any pain, and also remain in good health.20 Occlusion is generally classified into normal occlusion, ideal occlusion and malocclusion.

Ideal Occlusion: Ideal occlusion is defined as a condition when maxilla and mandible have their skeletal bases of correct size relative to one another, and the teeth are in correct relationship in the three spatial planes at rest.21 Other concepts relating to ideal occlusion in permanent dentition are ideal mesiodistal and buccolingual inclinations, correct approximal relationships of teeth, exact overlapping of upper and lower arch both laterally and anteriorly, existence of mandible in position of centric relation, and also presence of correct functional relationship during mandibular excursions.22

Normal Occlusion: Angle defined normal occlusion as the occlusion when upper and lower molars were in relationship such that the mesiobuccal cusp of upper molar occluded in buccal cavity of lower molar and teeth were all arranged in a smoothly curving line.23 Normal occlusion is an occlusion within accepted definition of the ideal and which caused no functional or aesthetic problems.22 Andrews proposed six distinct characteristics of normal occlusion, i.e., molar relation-
Measurement of dental arch curve using mathematical models: Study of dental arch characteristics aids in the early detection of future orthodontic problems. Evaluation of the intra-arch spacing leads to the forecasting and prevention of ectopic or premature teeth eruption. Mathematical modelling and analysis during primary dentition plays a major role in assessing the arch dimensions and spacing and helps to guide the development of a proper alignment in permanent dentition during the mixed dentition period. The prediction of arch variations and state of occlusion during primary and mixed dentition are crucial for establishing ideal desired esthetic and functional occlusion in permanent dentition.

The first proposed models were qualitative than quantitative i.e., ellipse, parabola, conic section, etc when describing the human dental arch and were attempted to explain mathematically the human dental arch in terms of polynomial equations of different orders. The main drawback of those theory was that it could not explain asymmetrical features or predict all forms of the arch. Later contributions to dental literature was made by different studies using teeth of various sample populations children in general, and a mathematical analysis of the dental arch in particular.

While some authors relied on symmetrical features of dental curvature, others utilized the asymmetrical cubic splines to describe the dental arch. BeGole assumed the cubic spline appropriately represented the general maxillary arch form of persons in normal occlusion. He developed a FORTRAN program on the computer to interpolating different cubic splines for each subject studied. This was in contrast to the view that the dental arch form through a set of quadratic equations and a generalized quadratic equation for defining the dental arch. Braun expressed the beta function by means of a mathematical equation. The beta function was a symmetrical function and did not explain observed variations in form and shape in actual human samples. It was observed that 4th order polynomials were actually a better fit than the splines, in later analyses in the 1990s, it appeared that these were even better than the beta. During twenty century the dental curve was expressed as a 3-D structure and conducted studies to know the 3-D inclinations of the dental axes, assessing arch curves of both adolescents and adults.

AlHarbi studied the dental arch curvature of individuals with normal occlusion formed the opinion that the 4th order polynomials could be effectively used to define a smooth dental arch curve which could further be applied into fabricating custom arch wires or a fixed orthodontic apparatus, which could substantially aid in dental arch reconstruction or even in enhancement of esthetic beauty in patients.

Comparison between Dental Arch Analysing models: Dental arch plays a key role in the satisfactory diagnosis, in orthodontic treatment, has emerged as an important part of modern dentistry. The early literature shows use of conventional anatomical points on incisal edges and on molar cusp tips and mathematical forms like ellipse, parabola, cubical spline, etc and geometrical shapes like catenary to classify forms of the dental arch.

Hayashi used mathematical equations of the form: $y = ax^n + e^{(x^2-b^2)}$ and applied them to anatomic landmarks on buccal cusps and incisal edges of numerous dental casts. However, the method was complex and required estimation of the parameters like $a$, $b$, etc. Also, Hayashi did not consider the asymmetrical curvature of the arch.

Fourth degree polynomial for defining the dental arch curve was introduced by Liu and a generalized quadratic equation for studying the close fit of shapes like the parabola, hyperbola and ellipse for describing the form of the human dental arch by Biggerstaff. Later sixth degree polynomials ensured a better curve. Many authors used a parabola of the form $x^2 = -2py$ for describing the shape of the dental arch compared to the catenary curve form defined by the equation $y = (e^x + e^{-x})/2$. The equation $(x^2/b^2) + (y^2/a^2) = 1$ defines an ellipse.

A computer program developed in FORTRAN was used to interpolate a cubic spline for individual subjects who were studied to effectively find out the perfect mathematical model to define the human dental arch and the splines used in analysis were either symmetrical or asymmetrical. A finite element analysis used in a different method to compare dental-arch forms was affected by homology function and has lack of element design. A multivariate principal component analyses developed to determine size and shape factors from numerous linear measurements could not satisfactorily explain major variations in dental arch forms and the method failed to provide for a larger generalization in explaining the arch forms.
Dental Arch Curve analysis in Children with Normal Occlusion: In general the eruption of primary dentition in children usually follows a fixed pattern. The time of eruption of various teeth follow a definite pattern over the growing years of child. The differences of teeth forms, shape, size, arch spacing and curvature, etc. That characterize a given sample under study for mathematical analysis, also essentially vary with the nationality and ethnic origin of a child.30

In one longitudinal study, children of Scandinavian origin with normal occlusion, it was found that when children pass from adolescence into adulthood, a significant lack of stability in arch form was discernible.34 The dental arches in some children were symmetrical, while in others this was not so, indicating that symmetrical form of a dental arch was not a prerequisite for normal occlusion. All these studies, based on mathematical analysis of one kind or another, have thrown up substantial data. But the studies have not been able to achieve a high degree of correlation so as to deliver a generalized theory that can satisfactorily associate a single mathematical model for all dental arch forms in children with normal occlusion.30

Eventhough different research studies point to different mathematical models as better fit for defining the human dental arch, Biggerstaff chose the quadratic equation i.e., parabola, hyperbola and ellipse13 compared to the 4th degree polynomial, as adequately representing such dental arch curves.28 Pepe found the catenary curves inferior as a fit than even the quadratic equations, but also found the 6th order polynomials a far better fit than the 4th order polynomials.17

The research mentioned was thus neither conclusive nor able to perfectly define the shape and size of any and every dental arch. Based on the previous research on arch curve definition BeGole15 stated that, ideally, the arch form could be represented by a curve that had immense flexibility; this would then enable better fit on any dental arch, which, throughout child hood and youth, appeared changing and hence could also be altered by dental intervention methods. According to BeGo le,15 the cubic spline, when chosen as a mathematical dental form, also effectively eliminates the limitation of straight lines when fit to the posterior segments of the arch and hence offers various advantages in being used for defining the human dental arch, as compared with other methods.

This review presents the assumptions, circumstances and limitations of each study, as well as an overall picture of the current stage of research in the field. BeGole, Pepe, Biggerstaff, Lu and others have no doubt contributed substantially to research on the dental arch and the defining of the dental arch12,15,17,28 but even till the present, no study has been able to conclusively establish the shape of the human dental arch as a universal fit for all type of subjects.

It also arrives at the conclusion that further research, particularly by using modern imaging techniques and computer simulation could well provide for a better mathematical model for defining the dental arch of children in normal occlusion.

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